

Application of spherical Slepian functions to aeromagnetic data and in crustal field modelling

Ciaran Beggan¹ (ciar@bgs.ac.uk) and Kathy Whaler²

¹British Geological Survey, West Mains Road, Edinburgh EH9 3LA, United Kingdom

²School of GeoSciences, University of Edinburgh, UK

Models of the crustal magnetic field are typically represented using *spherical harmonic* coefficients. Rather than spherical harmonics, *spherical Slepian* functions (hereafter just Slepian functions) can be employed to produce a locally and also globally orthogonal basis in which to optimally represent the data in a region up to a given degree. The region can have *any* arbitrary shape and size [Ref 1].

In this poster we show some of the possible applications of Slepian functions to aeromagnetic data studies:

- optimally separate a crustal field model into its oceanic and continental regions in order to investigate the spectral content (Figures 1 and 2)
- compactly describe regional spherical harmonics in a sparse manner (Figure 3)
- reconstruct a smooth function from a series of input data points (Figure 4)

Spatial and spectral decomposition

Slepian functions can be tailored to be either band- or space-limited, allowing a trade-off between spectral and spatial concentration in the region and leakage beyond. It is only necessary to solve for N Slepian coefficients to optimally concentrate the energy of the Slepian functions into the region of interest. $N = (L+1)^2 R/4\pi$, where N is the so-called Shannon Number, L is the maximum spherical harmonic degree and R is the area of the region as a fraction of the full sphere.

We optimally separate the spherical harmonic coefficients of a crustal field model, MF7 [2], into its oceanic and continental regions in order to investigate the spectral content of each part [3]. The spectral power of each region is examined over degrees $L = 16$ –133 (Figure 1). We compare the resulting oceanic spectrum to a forward Vertically Integrated Magnetisation model [4] to check the decomposition is reasonable (Figure 2).

Similar analyses can be applied to smaller regions (e.g. aeromagnetic surveys).

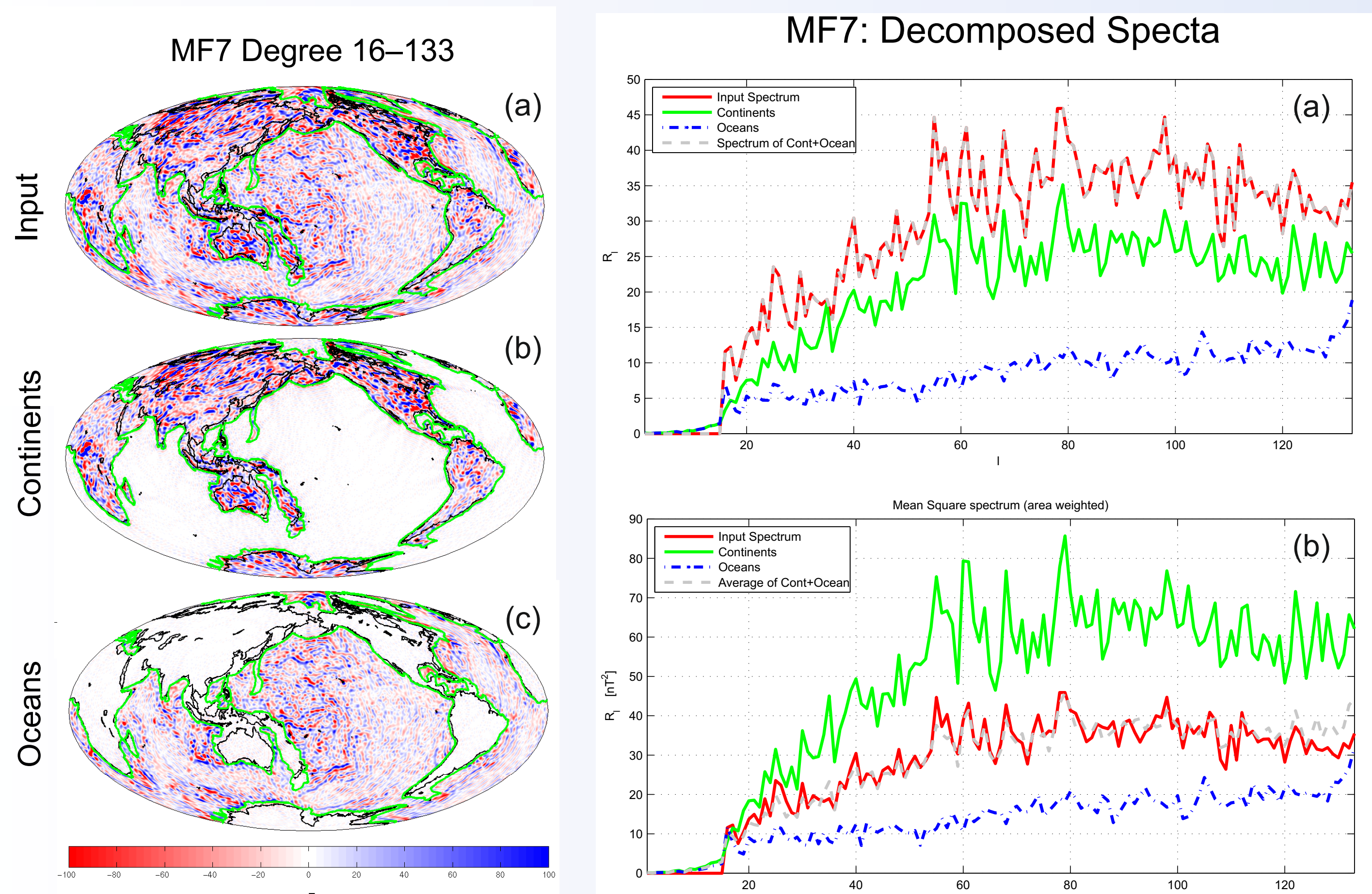


Figure 1: Decomposition of the MF7 crustal field model. Left panels show the radial component of the field over degree $L = 16$ –133: (a) input; (b) continental region; (c) oceanic region. The boundary between the continental and oceanic crust is outlined in green and includes submarine continental shelves. Right panels show the spectral decomposition: (a) unweighted spectra; (b) area-weighted spectra. Note there is some 'leakage' between the regions, as the trade-off can never be perfect.

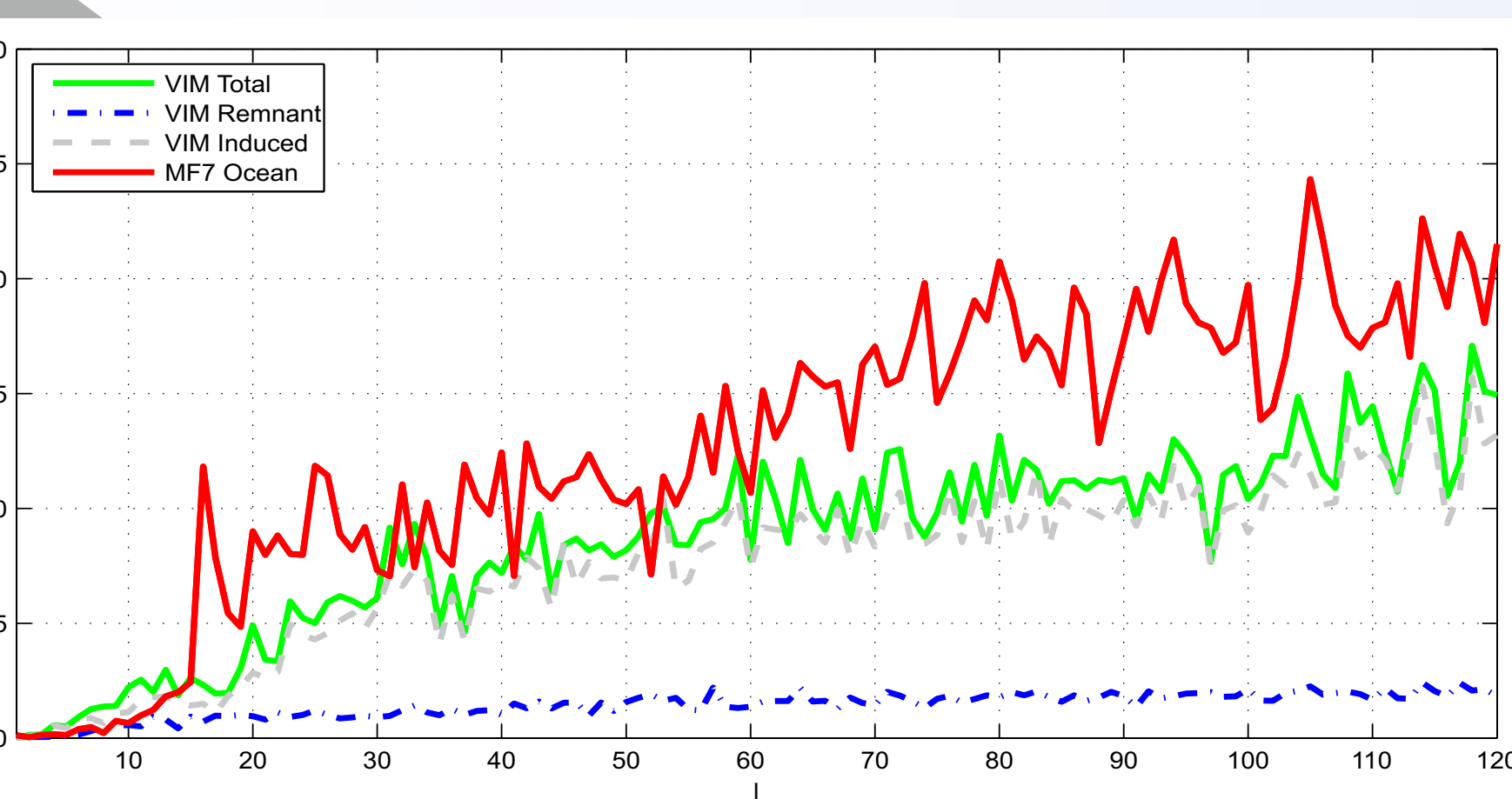


Figure 2: Comparison of the MF7 area weighted oceanic spectrum (red line) to the VIM models of Masterton *et al.* (2013) up to degree $L = 120$. For the VIM model, the blue line is the remnant magnetisation of the ocean crust, grey line is the induced magnetisation and the green line shows the total magnetisation strength per degree. The models match reasonably well considering they are computed from completely different datasets.

Approximation of a regional model

It is possible to use Slepian functions to describe spatially-limited areas in an efficient manner. The upfront cost comes from computing the basis functions (only once). Figure 3 shows how to compactly represent the magnetic field for an area of Africa.

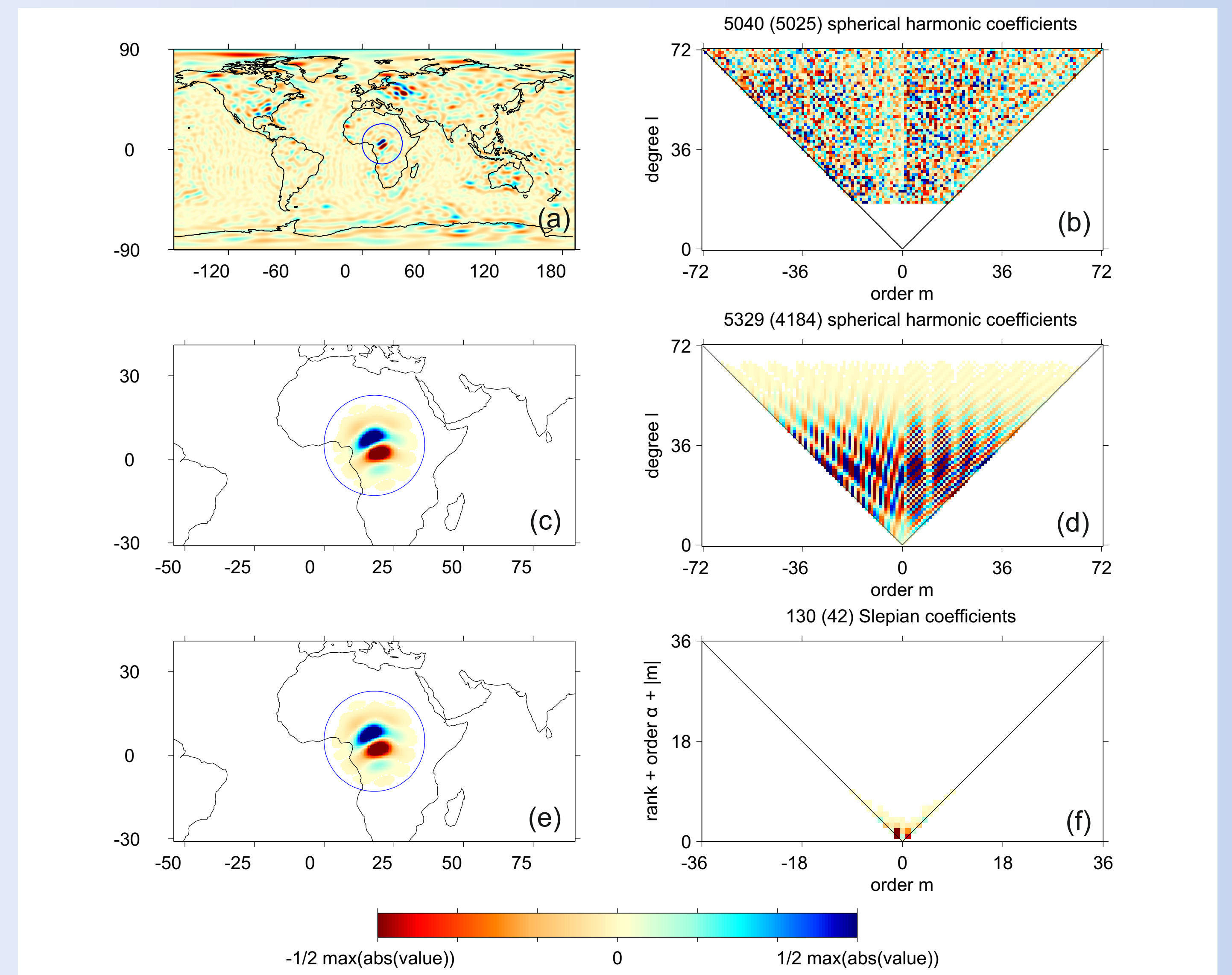


Figure 3: Global and local representations of the lithospheric magnetic field in the spherical-harmonic and Slepian bases. Negative/positive m indicates coefficients multiplying $\sin/\cos(m \cdot \phi)$, where ϕ is longitude. (a) Map of the radial component of the internal magnetic field at the Earth's surface according to the POMME4 model [5] bandpassed between spherical harmonic degrees 17 and 72, and (b) the spherical harmonic coefficients themselves. In total 5040 coefficients are needed to represent the global field, of which 5025 exceed a "1/1000" significance threshold of one thousandth of the maximum absolute value of all coefficients. Values below this relative threshold are left white in this and panels (d) and (f). (c) Map of the 'Bangui anomaly', a highly localized feature in central Africa. The anomaly was obtained by multiplying the radial field, low-passed to degree 36, by the Slepian function of bandwidth 36 that is best concentrated to the circular area of radius = 18° , shown in blue. The resulting localized field is now low-passed as can be seen in (d). Of the 5329 coefficients necessary to represent this anomaly, 4184 exceed the "1/1000" threshold.

(e) An approximation of the same anomaly using the $N = 130$ best-localized of the 5329 Slepian functions concentrated in the region. Of the 130 coefficients only 42 exceed the "1/1000" threshold, as shown in (f), which has a truncated axes. The approximation in the region of interest is beyond reproach and the representation by the Slepian, compared to the spherical harmonics, expansion is truly sparse. Hence, large datasets can be represented very compactly using Slepian functions.

(f) An approximation of the same anomaly using the $N = 130$ best-localized of the 5329 Slepian functions concentrated in the region. Of the 130 coefficients only 42 exceed the "1/1000" threshold, as shown in (f), which has a truncated axes. The approximation in the region of interest is beyond reproach and the representation by the Slepian, compared to the spherical harmonics, expansion is truly sparse. Hence, large datasets can be represented very compactly using Slepian functions.

Reconstruction using sparse data

Slepian functions can be used to reconstruct a smooth function from a limited set of data. Figure 4 shows how the radial magnetic field can be reconstructed from a sparsely sampled set of data. The misfit decreases with increasing numbers of input data.

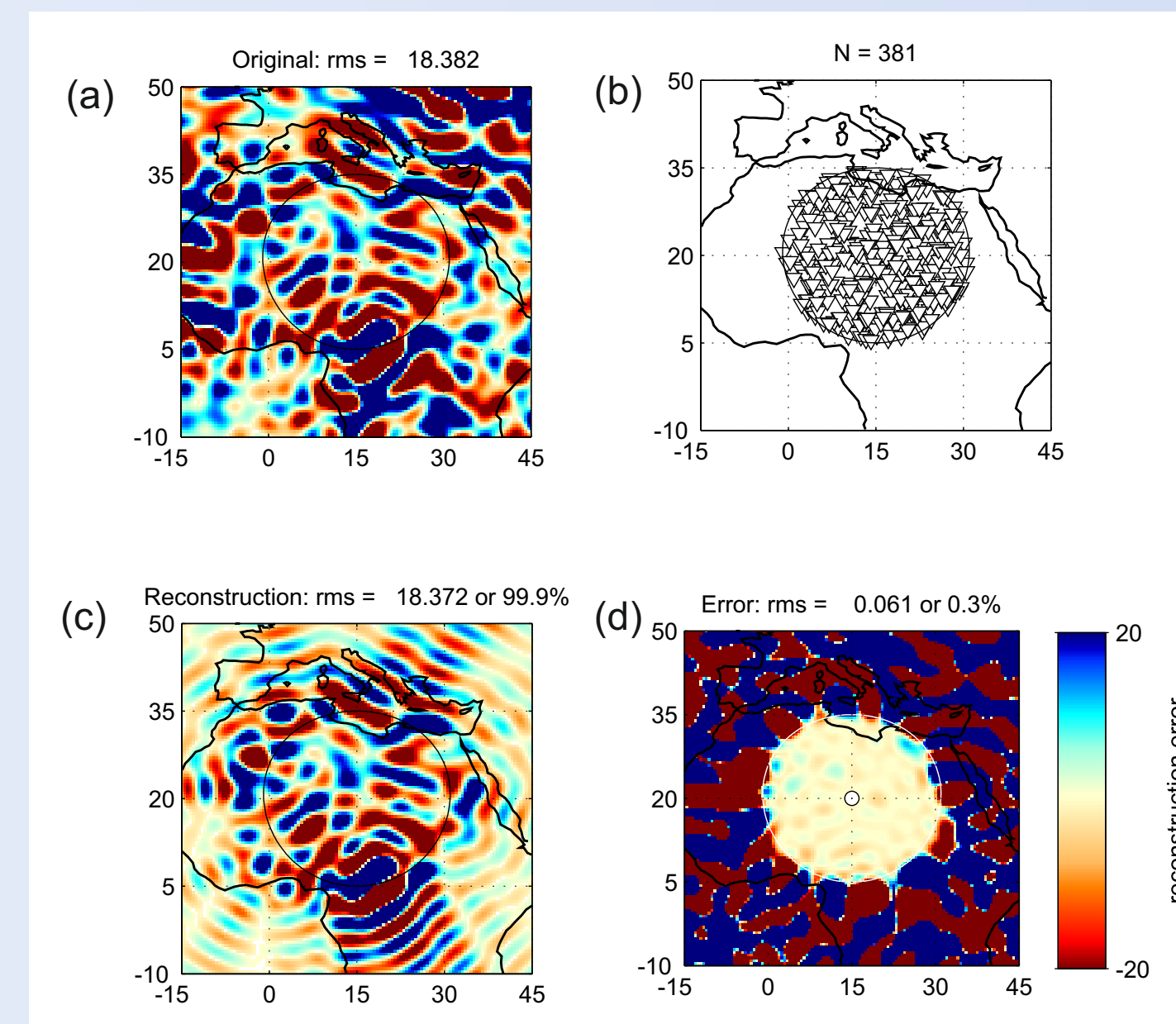


Figure 4: Efficient reconstruction of sparsely sampled data, illustrated using a bandlimited model of the crustal field from the NGDC-720 model [6] over degree $L = 17$ –72. The input data are shown in (a). The function is randomly sampled within the circular region of interest (radius: 15°) with N points, as displayed in (b). Reconstruction is achieved by generating Slepian eigenfunctions for the region and solving for a set of Slepian coefficients from the sampled data using Singular Value Decomposition. The Slepian eigenfunctions are multiplied by the solution coefficients and summed to form the reconstruction in (c). The difference between the input data and the reconstruction is shown in (d). The *rms* metric is the root mean square of the pixel values within the region. This example shows the method works well close to or over a large anomaly (Bangui). A similar experiment with spherical harmonic functions would generate a very large number of coefficients (i.e. like in Figure 3).

Discussion

Slepian functions can be effectively used in aeromagnetic applications for storage and analysis of data sets. Like spherical harmonic modelling, Slepian functions retain the long wavelength parts of the signal. Hence, they do not work quite so well on restricted bandwidths.

References

- [1] Dahlen, F. A. and F. J. Simon (2008). Spectral estimation on a sphere in geophysics and cosmology, *Geophys. J. Int.*, **174**, 774–807.
- [2] Maus, S., H. Lühr, M. Rother, K. Hemant, G. Balasis, P. Ritter, and C. Stolle (2007). Fifth-generation lithospheric magnetic field model from CHAMP satellite measurements, *Geochim. Geophys. Geosyst.*, **8**, Q05013.
- [3] Beggan, C., Saarikari, J., Whaler, K. and F. Simons (2013). Spectral and spatial decomposition of lithospheric magnetic field models using spherical Slepian functions, *Geophys. J. Int.*, **193**, 136–148.
- [4] Masterton, S. M., D. Gubbins, R. D. Müller and K. H. Singh (2013). Forward modelling of oceanic lithospheric magnetization, *Geophys. J. Int.*, **192**, 951–962.
- [5] Maus, S., M. Rother, C. Stolle, W. Mai, S. Choi, H. Lühr, D. Cooke, and C. Roth (2006). Third generation of the Potsdam Magnetic Model of the Earth (POMME), *Geochim. Geophys. Geosyst.*, **7**, Q07008.
- [6] Maus, S. (2010). An ellipsoidal harmonic representation of Earth's lithospheric magnetic field to degree and order 720, *Geochim. Geophys. Geosyst.*, **11**, Q06015.